

# C.U.SHAH UNIVERSITY

## Summer Examination-2016

Subject Name : Numerical Methods

Subject Code : 4SC04MTE1

Branch: B.Sc.(Mathematics, Physics)

Semester : 4

Date : 16/05/2016

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions (14)**
- a) Write Simpson's one third rule. (01)
  - b) Write  $n^{th}$  approximation of Newton-Raphson method. (01)
  - c) Give value of  $f'(x)$  by Newton's forward interpolation formula. (01)
  - d) What is the value of  $h$ ? (01)
  - e) Newton-Raphson method has second order convergence. Is following statement is true or false? (01)
  - f) Write Picard's formula for  $\frac{dy}{dx} = f(x, y)$ . (01)
  - g) How many ways we can find the solution of  $\frac{dy}{dx} = f(x, y)$  with  $f(x_0) = y_0$ . Give any two names of method. (01)
  - h) What is the value of  $f''(x)$  in general by Stirling interpolation formula? (01)
  - i) Find the values of  $\int_0^1 e^x dx$  with  $h = 1/2$  by Trapezoidal rule. (02)
  - j) Give value of  $a$  &  $b$  such that root of  $f(x) = 0$  lies between  $a$  &  $b$ , where  $f(x) = x^2 + x - 5$ . (02)
  - k) Give general formula for Modified Euler Method. (02)

**Attempt any four questions from Q-2 to Q-8.**

- Q-2 Attempt all questions (14)**
- a) Find a root of  $f(x) = 3x - 6 - \log_{10} x$  using Iteration Method up to four decimal places. (07)
  - b) Evaluate  $\int_0^{10} e^x dx$  by Weddle's rule with  $h = 1$ . (07)
- Q-3 Attempt all questions (14)**
- a) Prove that Newton-Raphson Method has second order convergence. (07)
  - b) Using Picard's Method, obtain solution up to the fifth approximation to the equation  $\frac{dy}{dx} = x + y; y(0) = 1$ . Also find  $y(0.1)$ . (07)



- Q-4** **Attempt all questions** (14)
- a) Given  $y' = x^2 + y^2$ ,  $y(0) = 1$ . Determine  $y(0.1)$  and  $y(0.2)$  by Modified Euler Method. (07)
- b) Find a positive root of  $f(x) = x - \cos x$  by False Position Method correct to three decimal places. (07)

- Q-5** **Attempt all questions** (14)
- a) Derive  $f'(x)$  by Newton's Forward Interpolation Formula. (07)
- b) Compute  $y(2)$  if  $y(x)$  satisfies the equation  $\frac{dy}{dx} = \frac{1}{2}(x + y)$  given  $y(0) = 2$ ,  $y(0.5) = 2.636$ ,  $y(1) = 3.595$  and  $y(1.5) = 4.968$ . Using Milne's Method. (07)

- Q-6** **Attempt all questions** (14)
- a) Find  $y'(x)$  and  $y''(x)$  at  $x = 6$ . For the function  $y = f(x)$  given in the table: (07)
- |     |        |        |        |        |        |        |
|-----|--------|--------|--------|--------|--------|--------|
| $x$ | 1      | 2      | 3      | 4      | 5      | 6      |
| $y$ | 2.7183 | 3.3210 | 4.0552 | 4.9530 | 6.0196 | 7.3891 |
- b) Obtain approximate value of the root of  $x = \sin x + \frac{\pi}{2}$  by Graphical Method. (07)

- Q-7** **Attempt all questions** (14)
- a) Compute  $f'(0.8)$  and  $f''(0.8)$  using the following table: (07)
- |        |         |         |         |         |         |
|--------|---------|---------|---------|---------|---------|
| $x$    | 0.4     | 0.6     | 0.7     | 0.9     | 1.2     |
| $f(x)$ | 1.08107 | 1.18546 | 1.25517 | 1.43309 | 1.81066 |
- b) Derive Newton-Raphson Iteration formula for  $\sqrt[n]{R}$ . Also find the value of  $\sqrt[3]{5}$  using this formula. (07)

- Q-8** **Attempt all questions** (14)
- a) Using Runge-Kutta Method of fourth order, solve the following for  $y(0.1)$  and  $y(0.2)$  given that  $\frac{dy}{dx} = xy + y^2$ ,  $y(0) = 1$  with  $h = 0.1$ . (07)
- b) The velocity  $v$  of a particle at distance's from point on its path is given by the following table: (07)

$s$ (m)	0	10	20	30	40	50	60
$v$ (m/s)	47	58	64	65	61	52	38

Find the time taken to travel 60 meter. Using Simpson's one-third rule.

